CHAP: 2: MECHANICAL PROPERTIES OF FLUIDS.

* **Fluid:** It is a substance which can flow.
  This substance can deforms continually under the action of external force.
  Fluid is a phase of matter in the form of liquid, gases on plasmas.
  eg. Air, water, flour dough, toothpaste etc.

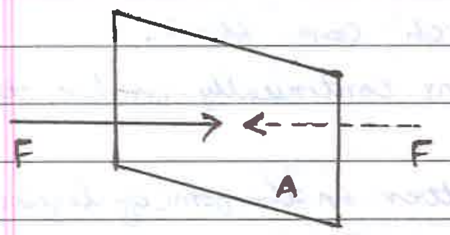
Q. State the properties of an ideal fluid.
→ 1) It is incompressible: It's density is constant
2) Its flow is irrotational: It's flow is smooth, there are no turbulences in the flow.
3) It is non viscous: There is no internal friction in the flow. (viscosity = 0)
4) Its flow is steady: It velocity at each point is constant in time

- Solids can be subjected to shear stress (tangential stress. as well as normal stress (compressive, tensile).
- Ideal fluids can only be subjected to normal (compressive) stress called as pressure. most of fluids offers very weak resistance for deformation. Real fluids display viscosity & so are capable of being subjected to low levels of shear stress.

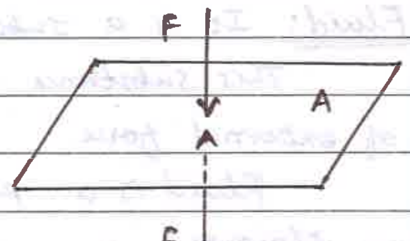
* **properties of Fluids:**
1) They do not oppose deformation, they get permanently deformed.
2) They have ability to flow.
3) They have ability to take the shape of the container.

* **Hydrostatics:** It is the branch of physics which deals with the properties of fluids at rest.
* **Pressure:** The normal force exerted by a fluid at rest per unit surface area of contact is called as pressure of the fluid.

$$\boxed{P = \frac{F}{A}}$$



ⓐ Fluid exert force on vertical surface



ⓑ Fluid exert force on horizontal surface.

fig ⓐ shows a fluid exerting normal force on a vertical surface & fig ⓑ shows a fluid exerting normal force on a horizontal surface.

An object having small weight can exert high pressure if its weight act on surface area.

eg. A force of 10N acting on $1\,cm^2$ results a pressure of $10^5 N/m^2$. & a force of 10N acting on $1\,m^2$ results a pressure of $10\,N/m^2$.

SI unit of pressure is N/m^2 or pascal (Pa). $[1 N/m^2 = 1 Pa]$.

pressure is scalar quantity having dimensions $[M^1 L^{-1} T^{-2}]$

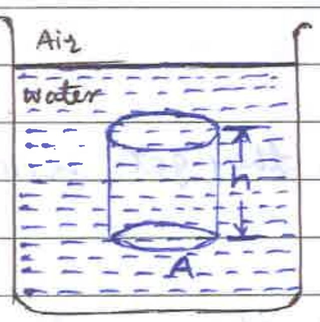
Also for pressure the diff. units are bar & torr.

$1\text{ bar} = 10^5 N/m^2$.

$1\text{ hectapascal } (hPa) = 100\,Pa$.

Q. Find the equation of pressure due to a liquid column

⟹



A vessel is filled with a liquid of density ρ. consider an imaginary cylinder of cross-sectional area A & height h inside the container. The liquid column exerts force on the bottom of cylinder is the weight of liquid column. $F = mg$ —①

∴ The pressure exerted by the liquid column on the bottom of cylinder is.

$$P = \frac{F}{A} \quad \text{—②}$$

put ① in ②

$$\therefore P = \frac{mg}{A} \quad \text{—③}$$

Now m = volume of cylinder × density of liquid

$$\therefore m = Ah\rho \quad \text{—④}$$

Put ④ in ③

$\therefore p = \frac{Ah\rho g}{A}$

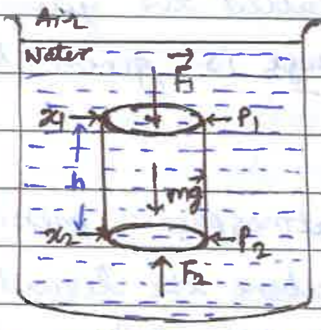
$\therefore \boxed{p = h\rho g}$ —⑤

Eq^n ⑤ gives the pressure due to a liquid of density ρ at a depth 'h' below the free surface of liquid.

* **Atmospheric pressure** :- Earts atmosphere is made up of a fluid, which exerts a downward force due to its weight. This pressure due to the force is called as atmospheric pressure.

$\therefore$ At any point the atmospheric pressure is the weight of a column of air of unit cross-section starting from that point & extending to the top of the atmosphere. So it is highest at the surface of the earth (at sea level) & it decreases as we go above the surface. At sea level it is called as normal atmospheric pressure. Beyond 8 km the density of air becomes negligible. So, atmospheric pressure at sea level can be taken to be 8 kms of air column above the level. In vacuum gas pressure is less that atmospheric pressure.

* **Absolute pressure and Gauge Pressure:**



Consider a tank filled with water as shown in fig. A imaginary cylinder of mass m & height h & area of base is A.

The vertical forces acting on the cylinder are.

1) Force $\vec{F_1}$ acts downwards at the top surface of cylinder, which is due to weight of water column above cylinder.

2) Force $\vec{F_2}$ acts upward at the bottom of the cylinder, which is due to the water below the cylinder.

3) The gravitational force mg, due to weight of water inside cylinders

At equilibrium state of cylinder

$F_2 = F_1 + mg$ —— ①

As P_1 & P_2 are the pressures at the top & bottom surfaces of the cylinder. $\therefore F_1 = P_1 A$ & $F_2 = P_2 A$.

also $mg = (\text{density} \times \text{volume})\, g$

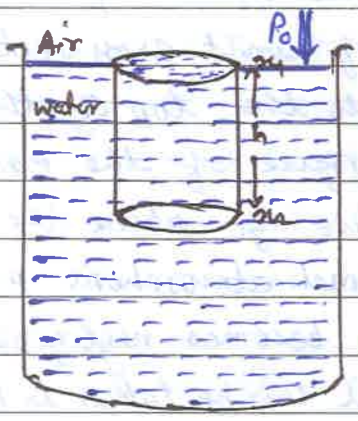
$mg = \rho A (x_1 - x_2) \cdot g$ —— ②

$\therefore P_2 A = P_1 A + \rho A g (x_1 - x_2)$ —— ③

$\therefore P_2 = P_1 + \rho g (x_1 - x_2)$ —— ③

Eq^n ③ used to find pressure inside a liquid & also atmospheric pressure.

To Find the pressure P at a depth 'h' below the liquid surface. let the top of an imaginary cylinder at the surface of liquid. Which is level x_1. & x_2 be a point at depth 'h' below the surface of liquid. As P_0 be the atmospheric pressure at the liquid surface.



$\therefore$ put $x_1 = 0$, $P_1 = P_0$, $x_2 = -h$ & $P_2 = P$ in eq^n ③.

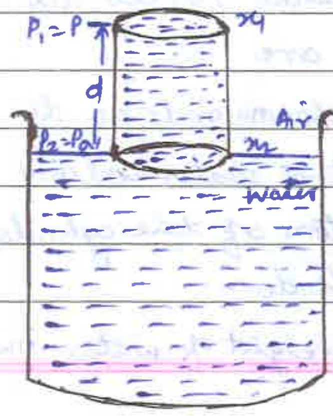
$\boxed{P = P_0 + h\rho g}$ —— ④

Eq^n ④ gives the total pressure. or the absolute pressure P, at a depth 'h' below the surface of the liquid.

Gauge Pressure :- The difference bet^n the absolute pressure and the atmospheric pressure is called the gauge pressure.

$\therefore$ From eq^n ④, Pressure gauge is given by

$\boxed{P - P_0 = h\rho g}$ —— ⑤



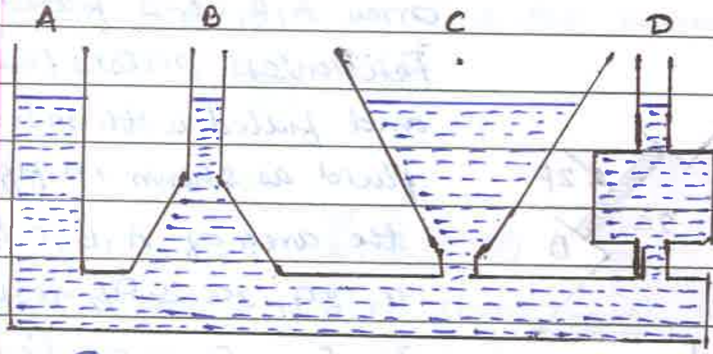
To find the atmospheric pressure at a distance d above the liquid surface is found from the fig. put $x_1 = d$, $P_1 = P_0$, $x_2 = 0$, $P_2 = P$ & $\rho = \rho_{air}$ in eq^n ③ $h = -d$.

$\boxed{P = P_0 - d\rho_{air}\, g}$ from eq^n ④

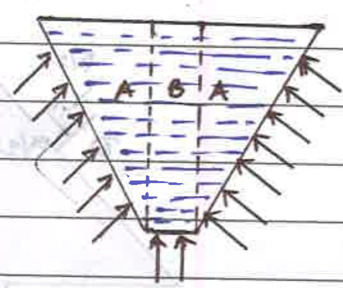
Q What is Hydrostatic Paradox? Explain it with suitable neat diagram.

⟹ <u>Hydrostatic Paradox</u>:-

The pressure exerted by liquid column of same height at the base of vessel does not depends upon the shape of ~~liq~~ vessel.



(a)



(b)

consider the inter connected vessels as shown in fig (a). when a liquid is poured in one vessel, it is observed that the level of liquids in all the vessels is the same. One can feel that the pressure of the base of vessel C would be more than that at the base of B & so liquid from vessel C would rise into B, but it is never observed. We know that the pressure at a point inside liquid depends only on the height of the liquid column above it - it does not depends upon the shape of vessels. As shown in fig (a) height of liquid column in each vessel is same. Therefore the pressure of liquid column in each vessel is the same when it is in equilibrium, so liquid in vessel C does not rise into B.

As shown in fig (b) the arrows indicates the forces exerted by the walls of vessel on the liquid column; which are ⊥er to walls of vessels at each point. These forces are resolved into vertical & horizontal components. The vertical components acts in the vessels in particular directions to balance the weight of liquid column & exerts the pressure at the base of the vessels.

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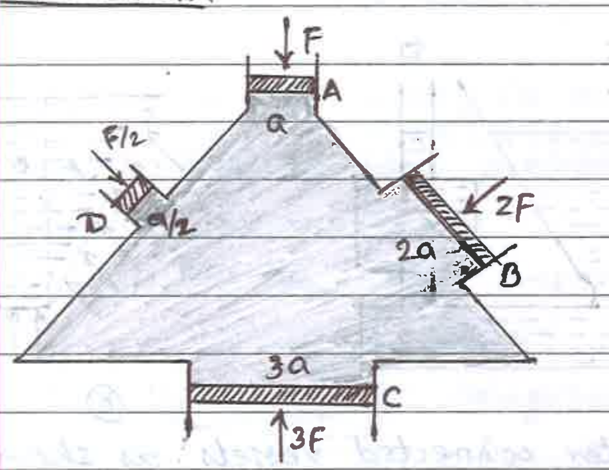
* **Pascal's Law:–** It states that the pressure applied at any point of an enclosed fluid at rest is transmitted equally & undiminished to every point of the fluid & also on the walls of the container, provided the effect of gravity is neglected.

Explanation:



Consider a vessel with four arms A, B, C & D fitted with Frictionless pistons (water tight) and filled with incompressible fluid as shown in fig. Let the area of A, B, C & D be a, $2a$, $3a$ & $a/2$ respectively. If force F is applied on the piston A, The pressure exerted on the liquid is $P = \frac{F}{a}$

It is observed that the other pistons B, C & D move outward; In order to keep these three pistons B, C & D in their original positions, The forces $2F$, $3F$ & $F/2$ respectively are required to be applied on the pistons, the pressures on B, C & D are.

on piston B ⇒ $P_B = \frac{2F}{2a} = \frac{F}{a}$

on piston C ⇒ $P_C = \frac{3F}{3a} = \frac{F}{a}$

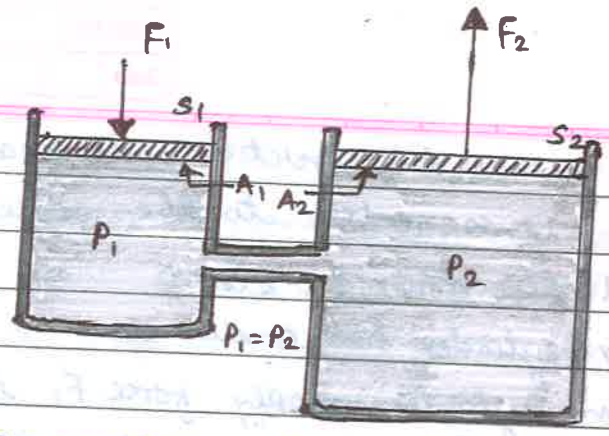
on piston D ⇒ $P_D = \frac{F/2}{a/2} = \frac{F}{a}$.

ie $P_B = P_C = P_D = P$. This indicates that the pressure applied on piston A is transmitted equally & undiminished to all parts of the fluid & the walls of the vessel.

Applications of Pascal's Law:–

1) **Hydraulic Lift:–** It is used to lift a heavy object using a small force; based on the principle of pascal's law.

As shown in fig. A tank containg a fluid is fitted with two pistons S_1 & S_2. S_1 has a smaller area of cross-section A_1 & S_2 has a much larger area of cross-section A_2 ($A_2 >> A_1$).



If we apply a force F_1 on the smaller piston S_1 in the downward direction it will generate pressure $P = \frac{F_1}{A_1}$ —① which will be transmitted undiminished to the bigger piston S_2. A force $F_2 = PA_2$ will be exerted upwards on it.

$$\therefore P = \frac{F_2}{A_2} \quad —②$$

From ① & ②

$$F_2 = F_1\left(\frac{A_2}{A_1}\right) \quad —③$$

F_2 is much larger than F_1. A heavy load can be placed on S_2 & can be lifted up or moved down by applying a small force on S_1.

2) Hydraulic brakes :- Hydraulic brakes are used to slow down or stop vehicles in motion, based on the same principle of hydraulic lift ie Pascal's law.

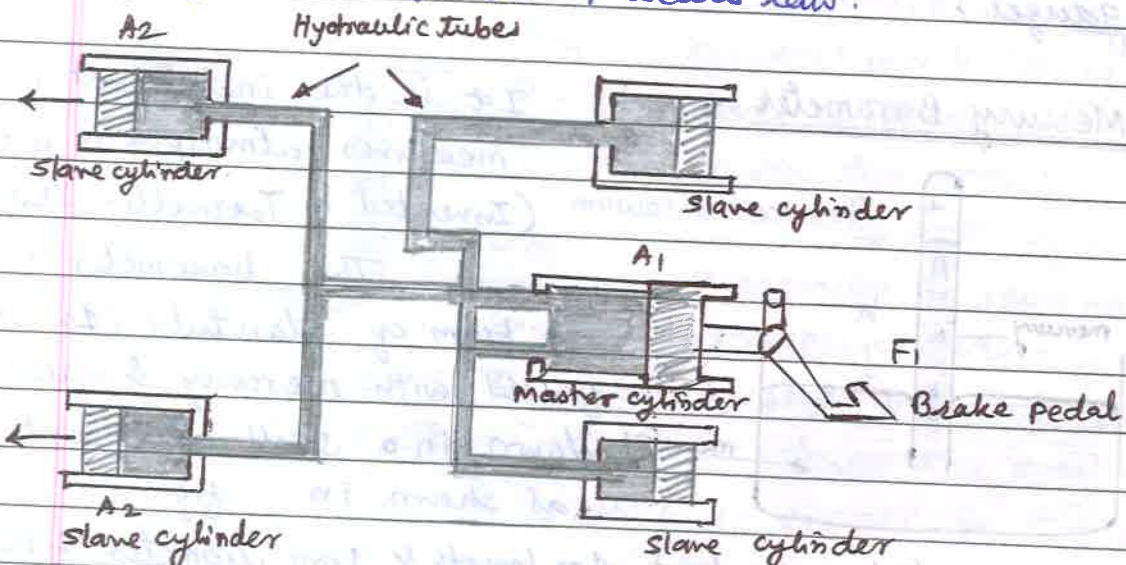


Figure shows schematic diagram of a hydraulic brake system. By pressing the brake pedal, the piston of the master cylinder is pushed in forward direction. As a result the piston in the slave cylinder which has much larger area of cross-section than that of master cylinder also moves in forward direction so as to maintain the volume of the oil constant

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The slave piston pushes the friction pads against the rotating disc, which is connected to the wheel. Thus moving vehicle will slow down or stop.

The master cylinder has smaller area of cross-section A_1 than A_2 of slave cylinder. Apply force F_1 to A_1, which generate pressure $P = F_1/A_1$. This pressure is transmitted undiminished throughout the system. The force F_2 is exerted on A_2 of slave cylinder.

$$\therefore F_2 = PA_2 = \frac{F_1}{A_1} \times A_2 = F_1\left(\frac{A_2}{A_1}\right)$$

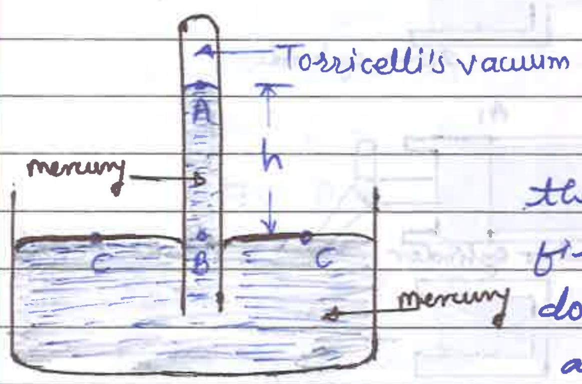
$\because A_2 >> A_1 \quad \therefore F_2 >> F_1$

Thus small force applied on the brake pedal gets converted into large force & slows down or stops a moving vehicle.

* Measurement of Pressure:-

The instruments used to measure pressure are called as pressure meters, pressure gauges or vacuum gauges.

A) Mercury Barometer:-



It is the instrument which measures atmospheric pressure. (Invented by Torricelli - Italian)

The barometer is in the form of glass tube (1m) completely filled with mercury & placed upside down in a small dish containing Hg as shown in fig.

1) A glass tube of about 1m length & 1cm diameter filled with Hg upto its brim. It is then quickly inverted into a small dish containg Hg. The level in the glass tube lowers as some mercury spills in the dish. A gap is created at the upper part of tube. This gap does not contain any air, it is called as Torricelli's vacuum. It contain some mercury vapors.

2) So the pressure at point A is zero $\therefore P_A = 0$
3) Consider a point C on the mercury surface in the dish & point B at same horizontal level inside the tube.
4) Pressure at C is atmospheric pressure P_0, because it is open to atmosphere. As points B & C are at same horizontal level $\therefore P_B = P_C = P_0$.
5) The point B is at a depth 'h' below the point A, as 'ρ' is the density of mercury.

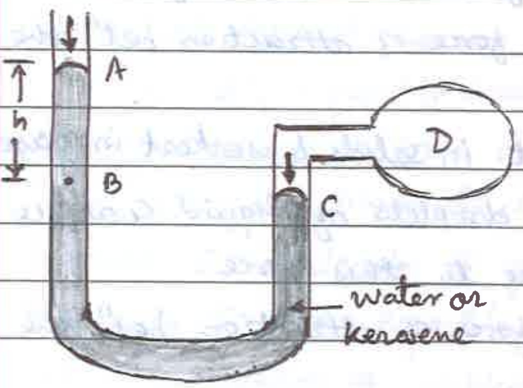
$\therefore P_B = P_A + h\rho g$ — but $P_A = 0$

$\therefore P_B = h\rho g$ ---- (P_{atm} = 76 cm of Hg = 760 mm of Hg)

(One torr = 1 mm of Hg)

$\therefore \boxed{P_0 = h\rho g}$, where h is height of mercury column in the mercury barometer.

B] <u>Open tube manometer:-</u> A manometer consist of U-shaped tube partly filled with a low density (ρ) liquid (water or kerosene) which having larger level difference bet^n two leves in the two arms.



One arm of the manometer is open to the atmosphere & other is connected to the container D; whose pressure 'P' is to be measured.

The pressure at A is atmospheric pressure P_0. To find pressure at C, which is exposed to the pressure of gas in the container, As points B & C are at same level. $\therefore P_C = P_B$. —— ①

but pressure at B is

$P_B = P_0 + h\rho g$ —— ②

where ρ = density of liquid & h is ~~hegy~~ height of liquid column above point B.

According to Pascal's principle $P_C = P_D$.

∴ The pressure P in the container is

$$P = P_C \quad —— ③$$

From eqⁿ ①, ② & ③

$$P = P_0 + h\rho g \quad —— ④$$

As the manometer measures the gauge pressure of the gas in the container D, The gauge pressure in the container D is

$$\boxed{P - P_0 = h\rho g} \quad —— ⑤$$

* <u>Surface Tension</u>:- It is defined as the force per unit length acting at right angles to an imaginary line drawn on the free surface of liquid. [$T = \frac{F}{L}$ N/m, dyne/cm $[T] = [M^1 L^0 T^{-2}]$]

* <u>Intermolecular force</u>:- It is the force betⁿ the molecules of substances.

There are two types of intermolecular forces.

1) <u>Cohesive force</u>:- It is the force of attraction betⁿ the molecules of same substance.

This force is strongest in solids & weakest in gases. So solids have definite shape. Small droplets of liquid coalesce into one and form a drop due to this force.

2) <u>Adhesive force</u>:- It is the force of attraction betⁿ the molecules of different substances.

* <u>Range of molecular force</u> (R): It is the maximum distance betⁿ molecules upto which the molecular force is effective.

$R = 10^{-9}$ m - so intermolecular forces are short range forces.

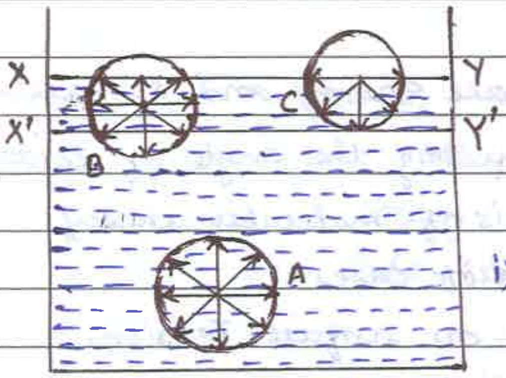
* <u>Sphere of influence</u>: It is an imaginary sphere drawn around the molecule with a molecule as a centre & radius equal to the range of molecular force

* <u>Surface Film</u>: It is the layer of liquid surface whose thickness is equal to the range of molecular force.

* **Free surface of a liquid**:- It is the surface of liquid which does not experience any shear stress.

Q. Explain Surface Tension on the basis of molecular theory (For the behaviour of liquid surface).

⟹



xy is the free surface of liquid & x'y' is the inner layer of liquid surface. consider three molecules A, B & C of liquid.

i) molecule A is entirely inside the liquid & it is surrounded by its nearest liquid molecules on all sides. Hence it is attracted by cohesive force on all sides. So resultent cohesive force acting on molecule A is zero. so molecule is free to move anywhere with the liquid.

2) molecule B lies within the surface layer & below the free surface of liquid. Larger part of its sphere of influence is inside the liquid & smaller part is in air. so strong downward cohesive force acts on liquid molecule. The adhesive force acting on molecule B due to air molecules above it. is weak. So molecule B gets attracted inside the liquid.

3) Molecule C lies exactly on the free surface of the liquid. It's half sphere of influence is in air & half in liquid. The adhesive force by air molecules on molecule C is much less than the cohesive force in downward direction. so that molecule C tries to pull ~~it~~ inside the liquid surface. So it gets attracted inside the liquid.

Thus all molecules in the surface film are acted upon by net cohesive force directed into the liquid. So that the surface film try to minimize its surface area & it remains under tension, known as surface tension. Hence the force due to surface tension acts tangential to the free surface of liquid.

* **Surface Energy:-** The surface molecules possess extra energy potential energy as compared to the molecules inside the liquid. This extra energy in the surface layer is called as surface energy.

surface energy = P.E. = workdone

Q.) Find the relation betn surface energy and surface tension.

- Angle of contact, Forces affecting the angle of contact.
- Angle of contact on the basis of molecular theory.
- shape of liquid drop & diff cases.
- Effect of impurity & temp. on surface Tension.
- pressure diff across the free surface of liquid.
- Excess pressure inside a liquid drop & liquid bubble ie Laplace's law of spherical membrane.
- Action of capillary
  - (a) Capillary fall
  - (b) Capillary rise
- Expression of surface tension by capillary rise method.

* **Hydrodynamics: (Fluids in motion):** It is a branch of physics which deals with the study of properties of fluids in motion.

* **Steady flow:** It is a flow of fluid in which the measurable properties like pressure and velocity at a given point are constant over time.

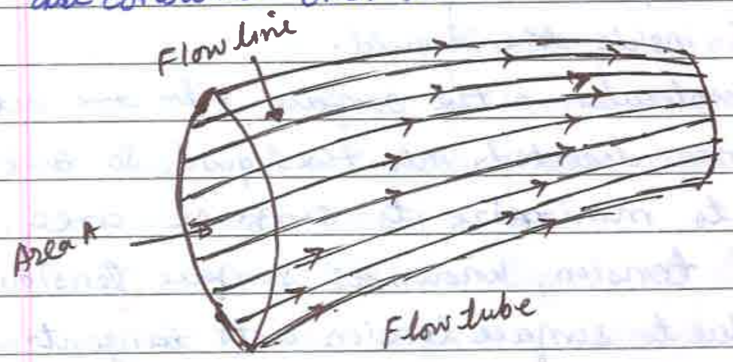
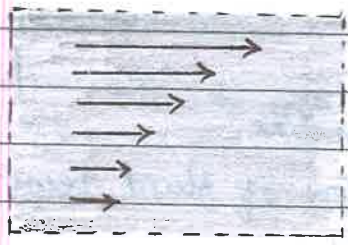


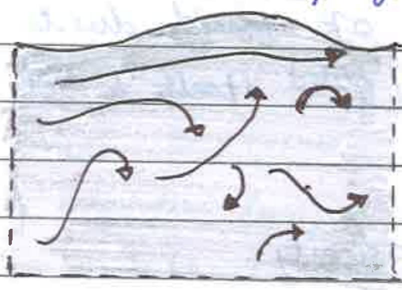
fig. Flow lines and Flow tube

* **Flow line:** It is the path of an individual particle in a moving fluid.
* **Streamline:** It is a curve whose tangent at any point in the flow is in the direction of the velocity of the flow at that point. [For steady flow streamlines & flow lines are identical].
* **Flow tube:** It is an imaginary bundle of flow lines bounded by an imaginary wall.
For a steady flow, the liquid can not cross the walls of a flow tube. Fluids in adjacent flow tubes can not mix.
* **Streamline flow (Laminar flow):-** It is a smooth flow of a liquid with velocity smaller than certain critical velocity.



• It is steady flow in which adjacent layers of a fluid move smoothly over each other.
• eg. steady flow of river.

* **Turbulent Flow:** It is an irregular & unsteady flow of liquid with a velocity greater than critical velocity.



• It is not steady flow and the flow pattern changes continuously
• eg. A flooded river flow or a tap running very fast.

Stream line flow	Turbulent flow
1) It is a smooth flow of liquid with velocity less than certain critical velocity.	1) It is an irregular & unsteady flow of liquid with a velocity greater than critical velocity
2) Velocity of fluid at a given point is always constant	2) velocity of fluid at a given point does not remain constant.
3) Two streamlines can never intersect & they are always parallel	3) At some points, the fluid may have rotational motion which gives rise to eddies.
4) stream line flow over a plane surface is divided into no. of plane layers.	4) A flow tube loses its order & particles move in random direction.

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* Critical velocity (V_c):- It is a velocity of fluid at which streamline flow is converted into turbulent flow.

* Reylonds number (R_n):- It is a pure number which indicates the type of flow of fluid.

The critical velocity of fluid is given by

$$V_c = \frac{R_n \eta}{\rho d} \quad \text{--- (1)}$$

where V_c = critical velocity, R_n = Reynolds number
η = coefficient of viscosity of fluid, ρ = density of fluid
d = diameter of tube.

From eq^n (1), Reynold's no. is

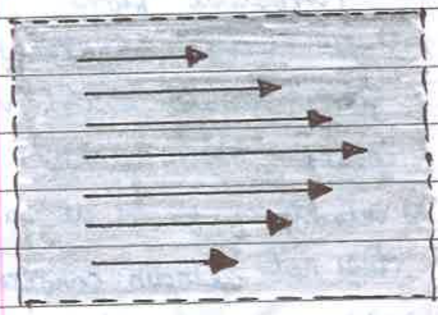
$$R_n = \frac{V_c \rho d}{\eta} \quad \text{--- (2)}$$

For R_n no unit & dimensions.

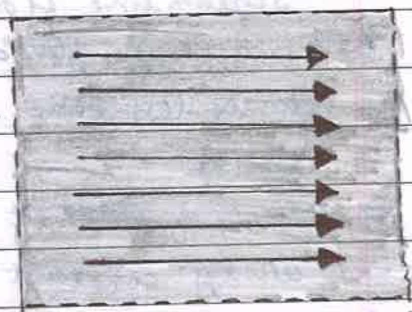
1) If $R_n < 1000$, The flow is streamline
2) If R_n is bet^n 1000 to 2000, the flow of fluid becomes unsteady, it streamline to turbulent flow.
3) If $R_n > 2000$, The flow of fluid is turbulent.

* Viscosity:- It is the property of fluid, due to which the friction is present within the fluid itself & bet^n the fluid & its surroundings.

- water has low viscosity
- syrup & honey has high viscosity.



(a) Viscous flow - Diff. layers flow with diff. velocities. Central layer flows fastest & outermost layers flow the lowest.



(b) Non-viscous flow. Diff layers flow with the same velocity.

In the flow of river water, it is found that the water near both sides of the river bank flows slow & as we move towards the centre of the river, the water flows faster gradully. At the centre, the flow is the fastest. ie there is some opposing force betn two adjacent layers of fluids which affects their relative motion.

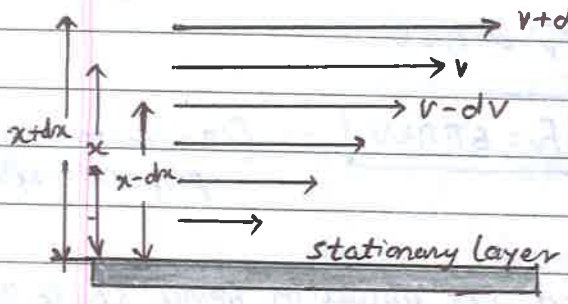
* Viscosity, viscous drag, velocity gradient.

Viscosity:- It is the property of fluid, by virtue of which it opposes the relative motion betn it's different (adjacent) layers.

Viscous drag: It is the tangential opposing force exerted by the adjacent layer to reduces its speed.

velocity gradient: It is defined as the rate of change of velocity with distance measured from a stationary layer.

$$\text{Velocity gradient} = \frac{dv}{dx}$$



$$\text{Velocity gradient} = \frac{\text{change in velocity bet}^n \text{ the layers}}{\text{Distance bet}^n \text{ two adjacent layers}}$$

$$= \frac{v+dv-v}{x+dx-x}$$

$$\therefore \boxed{\text{velocity gradient} = \frac{dv}{dx}} \text{ unit per sec.}$$

* State Newtons law of viscosity. Find the eqn of coefficient of viscosity & hence define it.

⇒ Newtons law of viscosity: For stream line flow the viscous force acting on any layer is directly proportional to the area of a layer and velocity gradient.

If F is the viscous force acting on a layer of surface area A & dv/dx is its velocity gradient

$$\therefore F \propto A \cdot \frac{dv}{dx}$$

$$\therefore F = \eta A \frac{dv}{dx} \quad \text{—(1)}$$ where η = Coefficient of viscosity of liquid.

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From eqⁿ ①

$$\boxed{\eta = \frac{F}{A\left(\frac{dv}{dx}\right)}} \quad \text{--- ②}$$

SI unit of $\eta = Ns/m^2$

$[\eta] = [M^1 L^{-1} T^{-1}]$

Coefficient of viscosity : It is defined as the viscous force per unit area per unit velocity gradient.

Q. state & Explain stoke's law with neat diagram.

→ stokes law :- It states that the viscous force acting on a spherical object falling through a viscous medium is directly proportional to the radius of the sphere, its velocity, and the coefficient of viscosity of the fluid.

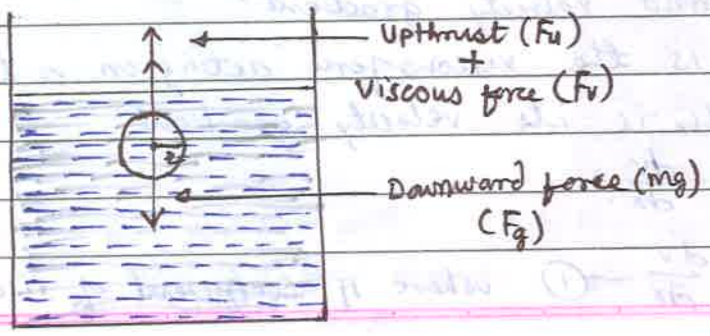
If r is radius of a sphere, falling through viscous fluid of coefficient of viscosity η with velocity v, so viscous force F_v is given by

$F_v \propto \eta r v$

$\therefore \boxed{F_v = 6\pi\eta r v}$ -- (6π = constant of proportionality)

Q Define terminal velocity. find its equation, Hence state the equation of coefficient of viscosity of liquid.

⟹ Terminal velocity : when a spherical body falling freely through a viscous fluid, initially its velocity increases & then it attains certain constant velocity, is called as terminal velocity.



consider a sphere of mass 'm', radius 'r' & density ρ falling under gravity through a viscous liquid (medium) of density 'σ' & coefficient of viscosity η. Initially velocity of sphere increases, but after some time it attains constant velocity, called as terminal velocity.

The diff. forces acting on the sphere are.

1) Viscous force (F_v), directed upward, it's magnitude increases with increase in its velocity.
2) Buoyant force or upthrust (F_U) directed upward.
3) Gravitational force (F_g) ie weight directed downwards.

F_g & F_U are constant, F_v increases with increase in velocity. at terminal velocity F_v also remains constant.

The net force acting on the sphere is

$$F = F_g - (F_v + F_U) \quad \text{--- (1)}$$

But at terminal velocity $F = 0$

$$\therefore F_g = F_v + F_U \quad \text{--- (2)}$$

$\therefore$ Total downward force = viscous force + Buoyant force

$$mg = 6\pi\eta r v + \frac{4}{3}\pi r^3 \sigma g \quad \text{------} \left(m = \frac{4}{3}\pi r^3 \rho \text{ - For sphere}\right)$$

$$\therefore \frac{4}{3}\pi r^3 \rho g = 6\pi\eta r v + \frac{4}{3}\pi r^3 \sigma g$$

$$\therefore 6\pi\eta r v = \frac{4}{3}\pi r^3 (\rho - \sigma) g$$

$$\therefore \boxed{v = \frac{2}{9}\frac{r^2 g(\rho - \sigma)}{\eta}} \quad \text{--- (3)}$$

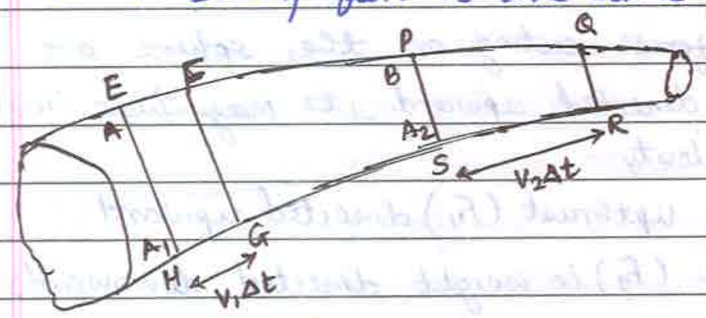
Eq^n (3) gives ~~initial~~ Terminal velocity of a sphere.

From eq^n (3), the coefficient of viscosity of liquid is

$$\boxed{\eta = \frac{2}{9}\frac{r^2 g(\rho - \sigma)}{v}} \quad \text{--- (4)}$$

Q What is equation of continuity ? Find this equation for steady flow of an incompressible fluid.

⇒ Equation of continuity :- The equation of continuity shows that the volume rate of flow of an incompressible fluid for a steady flow is the same throughout the flow.



Consider a steady flow of an incompressible fluid, the velocity of particles remains constant at a given point but it can vary from point to point. Consider sections A_1 & A_2. A_1 has larger cross-sectional area than A_2. Let v_1 & v_2 be the velocities of the fluid at A_1 & A_2 respectively; ~~because~~ $v_2 > v_1$ because a particle has to move faster in the narrower section as there is less space to accommodate particles behind it hence its velocity increases; it is only for incompressible fluid. The compressed fluid does not move faster through narrow space.

From fig sections A_1 & A_2 are at points A & B matter is neither created nor destroyed within the tube enclosed betn A_1 & A_2. So mass of fluid within this region remains constant over time.

Let V_1 be the speed of fluid at point A in time interval Δt, which crosses section EFGH. The volume of the fluid entering the tube at point A is $\rho A_1 v_1 \Delta t$. Similarly v_2 be the speed of fluid at point B in time interval Δt, which crosses section PQRS. The volume of the fluid entering the tube at B is $\rho A_2 V_2 \Delta t$.

∴ Mass of fluid in section EFGH = mass of fluid in section PQRS.

$$\therefore \rho A_1 V_1 \Delta t = \rho A_2 V_2 \Delta t$$

$\therefore \boxed{A_1v_1 = A_2v_2}$ or $Av = \text{constant}$

where Av is the volume rate of flow of a fluid.

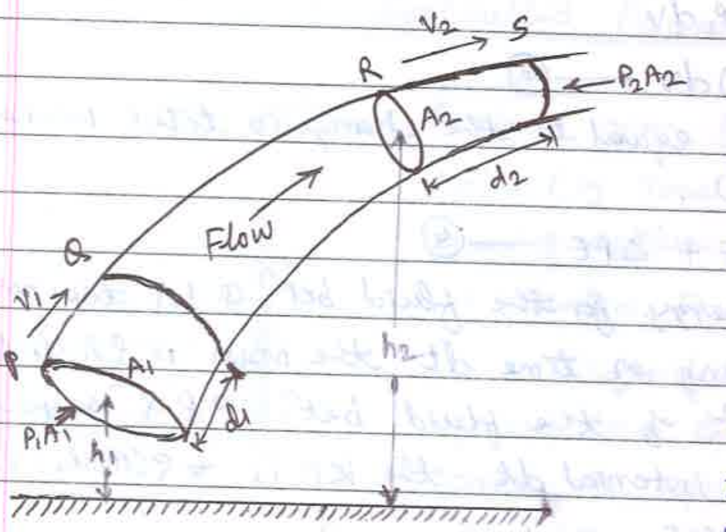
$\therefore Av = \frac{dV}{dt}$ where $\frac{dV}{dt}$ is the volume of fluid per unit time passing through any cross-section of the tube of flow.

it is also called as volume flux.

The term $\rho \frac{dV}{dt} = \frac{dm}{dt}$ is called as mass flux.

Q. State ~~and prove~~ Bernoulli's principle & find Bernoulli's equation with neat diagram.

$\Rightarrow$ Bernoulli's Principle: It is the phenomenon in which the speed of a fluid in a narrow region increases but the internal pressure of ~~the~~ a fluid in the same narrow region decreases.



Bernoulli's equation is related to the speed of fluid at a point, the pressure at that point & height of that point from the reference level.

To derive Bernoulli's eq^n we will prove that the net workdon on a fluid element by the pressure of the surrounding fluid is equal to the sum of the change in KE & change in gravitational P.E.

Fig. shown an ideal fluid of density 'ρ' through a tube of varying cross-section & height. consider an element of fluid that lies bet^n. cross-sections P & R.

Let v_1 & v_2 be the speeds of the fluid at lower end P & upper end R. A_1 & A_2 be the cross-section area at P & R. P_1 & P_2 be the pressures at P & R. d_1 & d_2 be the distances travelled by the fluid at P & R during time interval dt

$\therefore$ P_1A_1 & P_2A_2 be the forces acting on the equation of continuity. dV is the volume of fluid passing through any cross-section during time interval dt is the same.

ie $dV = A_1d_1 = A_2d_2$ —— (1)

There is no internal friction in the fluid as it is ideal so loss of energy due to friction is negligible (in water) so only non gravitational force does work on fluid element. which is due to pressure of the surrounding fluid. So net workdone on the element by the surrounding fluid during flow from P to R is

$W = P_1A_1d_1 - P_2A_2d_2$ —— (2)

(–ve sign is due to the force at R opposes the displacement of fluid.)

put (1) in (2)

$\therefore W = P_1dV - P_2dV$

$W = (P_1 - P_2)dV$ —— (3)

This workdone is equal to the change in total mechanical energy (KE+PE).

$\therefore W = \Delta KE + \Delta PE$ —— (4)

The mechanical energy for the fluid betn Q & R does not change

At the begining of time dt the mass is ρA_1d_1 & KE is $\frac{1}{2}\rho(A_1d_1)V_1^2$ of the fluid betn P & Q respectively.

At the end of interval dt the KE is $\frac{1}{2}\rho(A_2d_2)V_2^2$ betn section R & S.

$\therefore \Delta KE = \frac{1}{2}\rho(A_2d_2)V_2^2 - \frac{1}{2}\rho(A_1d_1)V_1^2$

$= \frac{1}{2}\rho dV V_2^2 - \frac{1}{2}\rho dV V_1^2$

$\therefore \Delta KE = \frac{1}{2}\rho dV(V_2^2 - V_1^2)$ —— (5)

Similarly at the beginning of time interval dt PE of mass m betn P & Q is $mgh_1 = \rho dVgh_1$ & at the end of dt. betn S & R is $mgh_2 = \rho dVgh_2$

$\therefore \Delta PE = \rho dVgh_2 - \rho dVgh_1$

$\therefore \Delta PE = \rho dVg(h_2 - h_1)$ —— (6)

From eq^n ③, ④, ⑤ & ⑥

$$(P_1 - P_2) dV = \frac{1}{2}\rho dV (V_2^2 - V_1^2) + \rho dV g (h_2 - h_1)$$

$$\therefore \boxed{(P_1 - P_2) = \frac{1}{2}\rho (V_2^2 - V_1^2) + \rho g (h_2 - h_1)} \quad — ⑦$$

Eq^n ⑦ is Bernoulli's equation.

It state that the workdone per unit volume of a fluid by the surrounding fluid is equal to the sum of the changes in kinetic & potential energies per unit volume that occur during the flow.

Eq^n-⑦ can be written as

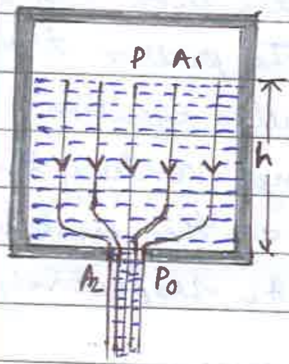
$$\boxed{P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2} \quad — ⑧$$

or

$$\boxed{P + \frac{1}{2}\rho V^2 + \rho g h = \text{constant.}} \quad — ⑨$$

* Applications of Bernoulli's Equation :-

1) Speed of efflux :- Efflux mean fluid out flow through small orifice (by Torricelli) It gives formula of speed of efflux from an open tank, which is identical to that of a freely falling body.



Consider a liquid of density ρ filled in a tank of large cross-sectional area A_1, having an orifice of cross-sectional A_2 ($A_2 << A_1$) at the bottom. Let V_1 & V_2 be the speeds of liquid at A_1 & A_2 respectively.

As inlet & outlet are exposed to the atmosphere ($\because P = P_0$)

'h' is the height of the free surface above the orifice.

$\therefore$ According to Bernoulli's eq^n for inlet & outlet

$$P_0 + \frac{1}{2}\rho V_1^2 + \rho g h = P_0 + \frac{1}{2}\rho V_2^2 \quad — ①$$

By eq^n of continuity $V_1 = \frac{A_2}{A_1} \cdot V_2$.

$$\therefore \frac{1}{2}\rho \left(\frac{A_2}{A_1}\right)^2 V_2^2 + \rho g h = \frac{1}{2}\rho V_2^2$$

$$\therefore \left(\frac{A_2}{A_1}\right)^2 V_2^2 + 2gh = V_2^2.$$

$\therefore 2gh = V_2^2 - \left(\frac{A_2}{A_1}\right)^2 V_2^2$

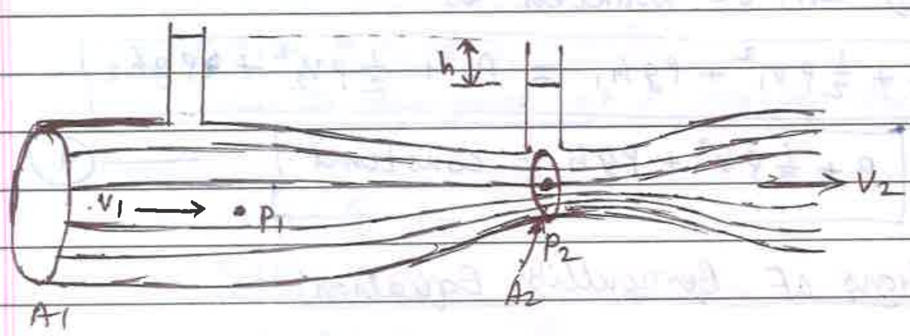
$2gh = V_2^2\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]$

If $A_2 << A_1$ $\therefore \left(\frac{A_2}{A_1}\right)^2$ is negligible

$\therefore \boxed{V_2 = \sqrt{2gh}}$ —— ②

Eq② gives velocity of efflux.

2) <u>Ventury tube</u>:



A ventury tube is used to measure the speed of flow of a fluid in a tube. The speed in the tube increases according to equation of continuity. The pressure thus decreases as required by the Bernoulli's equation.

A fluid of density ρ flows through ventury tube. A_1 is the area at wider part & A_2 at the construction. Let V_1 & V_2 be the speeds at A_1 & A_2 respectively.

$\therefore$ By Bernoulli's equation

$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$

$(P_1 - P_2) = \frac{1}{2}\rho(V_2^2 - V_1^2)$ —— ①

The height diff in vertical tubes of fluid is h.

$\therefore$ we have

$P_1 - P_2 = \rho g h$ —— ②

From ① & ②

$2gh = V_2^2 - V_1^2$ —— ③

The eqⁿ of continuity is $A_1V_1 = A_2V_2$ ∴ $V_1 = \frac{A_2}{A_1} \cdot V_2$
By putting value of V_1 or V_2 in eqⁿ ② gives rate of flow of fluid passing through a cross-section A_1 & A_2.

3) Lifting up of an aeroplane :-

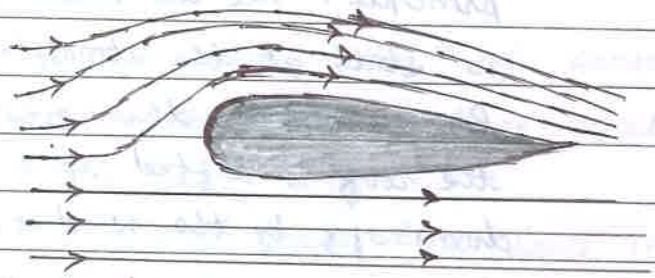


fig: Airflow along an aerofoil.

Fig shows cross-section of wings of an aeroplane. When an aeroplane runs on a runway, due to aerodynamic shape of its wings, the streamlines of air are crowded above the wings compared to those below it. Thus the air above the wings moves faster than that below the wings. According to Bernoulli's principle the pressure above the wings decreases & that below the wings increases. Due to this pressure diff; an upward force called the dynamic lift acts on the bottom of the wings of a plane. When this force becomes greater than the weight of aeroplane, the aeroplane takes off.

4) Working of an atomizer

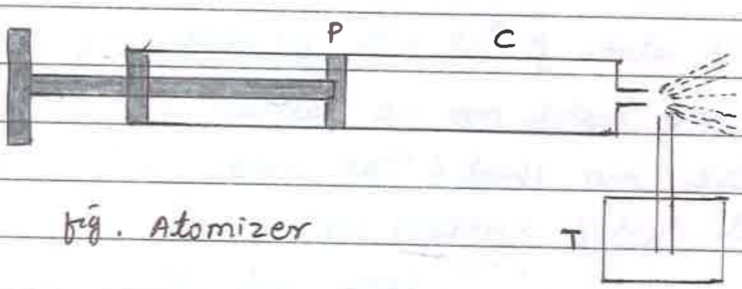
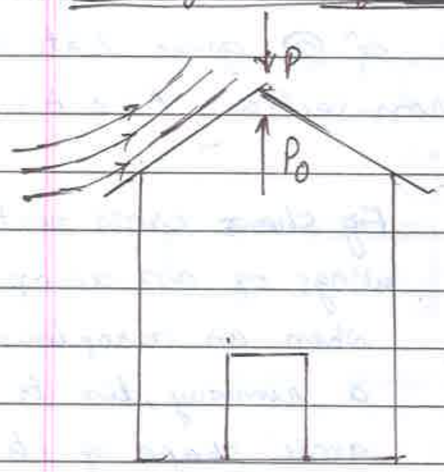


fig. Atomizer

The action of the Carburator of an automobile engine, paint gun, scent-spray or insect-sprayer is based on the Bernoulli's principle. In all these, a tube T is dipped in a liquid. Air is blown at high speed over the tip of this tube with the help of a piston 'P' in the cylinder 'C'. This high speed air creates low pressure over the tube, due to which the liquid rises in it and is then blown off in very small droplets with expelled air.

5) Blowing off of roofs by stormy wind:



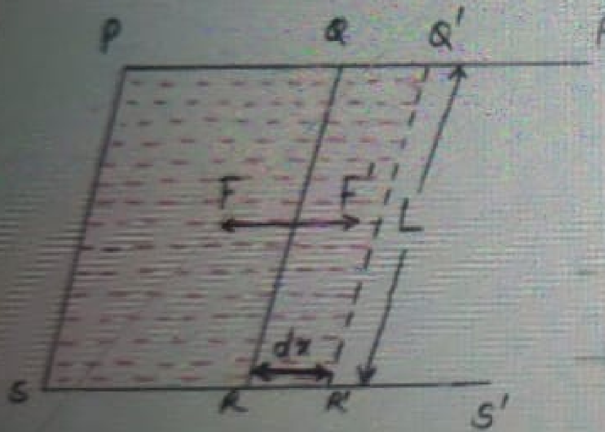
when high speed, stormy wind blows over a roof top, it causes low pressure P above the roof in accordance with the Bernoulli's principle. The air below the roof is still at the atmospheric pressure P_0. So due to this pressure diff, the roof is lifted up & is then down off by the wind as shown in fig

CHAP: 2 : Mechanical Properties of Fluids - Part - 3(B)

* **Surface energy**: The molecules in the surface layer possesses extra P.E. as compared to the molecules inside the liquid, is called as surface energy.

Que: Find the relation bet^n surface tension and surface energy.

→



The force acting on wire QR towards PS

$F = T \times 2L$ —— ①

(2 is due to two surfaces of soap film)

The workdone for displacement dx by F'

$dw = F' \cdot dx$ —— ②

but $F' = F = T \times 2L$

$\therefore dw = T \times 2L \cdot dx$ —— ③

but $dA = 2L \cdot dx$ - increase in surface area of soap film.

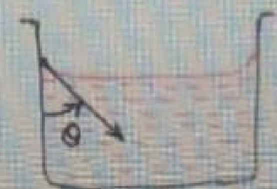
$\therefore dw = T \cdot dA$ —— ④

but $dw = P.E$ (extra in surface) = Surface energy.

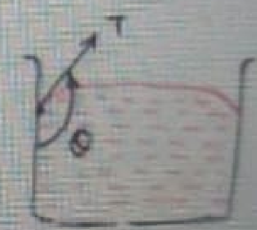
$\therefore \boxed{\text{Surface energy} = T \cdot dA}$ —— ⑤

Eq^n ⑤ gives relation bet^n surface tension & surface energy.

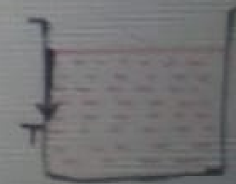
* **Angle of contact**: - It is the angle bet^n the tangent drawn to the free surface of liquid & the solid surface at the point of contact, measured within the liquid.



ⓐ liquid partially wets solid
($\theta < 90°$, acute)
Liquid surface is concave
kerosene in glass



ⓑ Liquid does not wets solid
($\theta > 90°$, obtuse)
liquid surface is convex
Mercury in glass



ⓒ Liquid completely wets solid
($\theta = 0$)
Liquid surface is straightline.
Clean water in glass

2

Factors on which the angle of contact depends. (characteristics of angle of contact)

1) The nature of solid & liquid in contact

2) Impurity present in liquid changes angle of contact

3) Temp. changes angle of contact

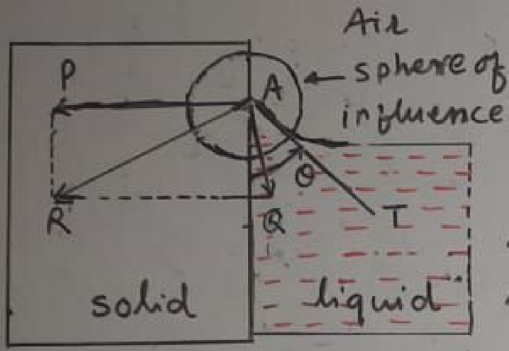
4) It remains constant for given solid & liquid pair

CHAP:2: Mechanical Properties of Fluids - Part - 4(A)

3

Que: Explain angle of contact on the basis of molecular theory.

⟹ 1) Concave Meniscus - Acute angle of contact



A = Liquid molecule

$\vec{AP}$ = Net adhesive force

$\vec{AQ}$ = Net cohesive force

$\vec{AR}$ = Resultent of $\vec{AP}$ & $\vec{AQ}$

AT = Tangent to liquid surface

θ = Angle of contact.

A liquid which partially wets solid are in contact (kerosene in glass). The net $\vec{AP} >> \vec{AQ}$. so resultent of $\vec{AP}$ & $\vec{AQ}$ is $\vec{AR}$ lies in solid which pulls liquid molecules downward at the point of contact so liquid meniscus becomes concave with $\theta < 90°$.

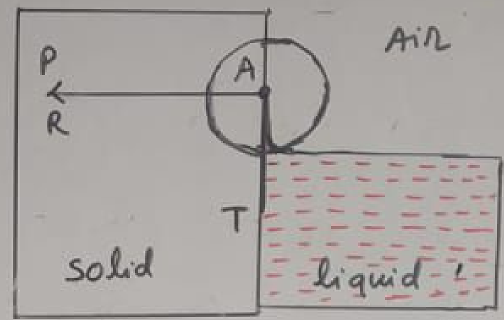
2) Convex meniscus - Obtuse angle of contact

A liquid does not wets solid [Hg in glass]

$\vec{AP} << \vec{AQ}$

∴ $\vec{AR}$ - lies inside liquid it push liquid molecules upward. So liquid meniscus becomes Convex with $\theta > 90°$.

3) Zero angle of contact



A liquid which completely wets solid [clean water in clean glass]

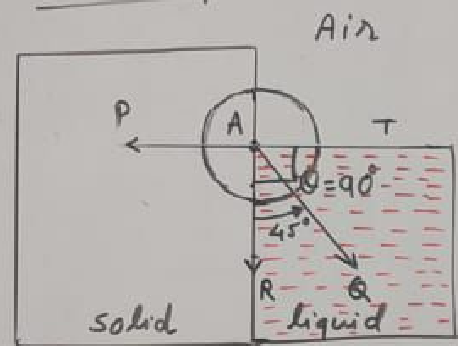
$\vec{AP} >>> \vec{AQ}$, $\vec{AQ} = 0$

(liquid molecules are very less at the point of contact).

∴ $\vec{AR}$ coincide with $\vec{AP}$.

so tangent AT is along the wall with $\theta = 0°$.

4) Angle of contact 90° & conditions For Convexity & concavity



Consider a hypthetical liquid having angle of contact 90° with a given solid. so net cohesive force $\vec{AC}$ of liquid is at 45° with the surfaces of solid & liquid.

So resultent $\vec{AR}$ is exactly vertical (along the solid surface).

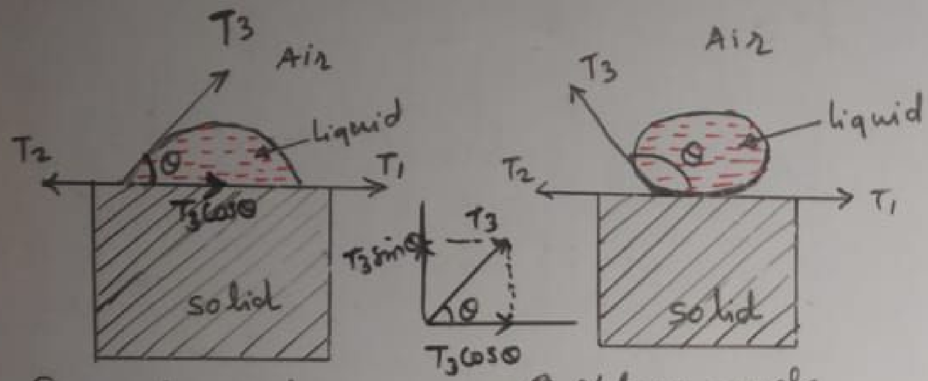
$\therefore \vec{AP} = \frac{AC}{\sqrt{2}}$. So the conditions for acute & obtuse angle of contact is

1) For acute angle of contact $\vec{AP} > \frac{AC}{\sqrt{2}}$

2) For obtuse angle of contact $\vec{AP} < \frac{AC}{\sqrt{2}}$.

CHAP: 2: Mechanical Properties of Fluids - Part - 4(B)

* Shape of liquid drops on solid surface



ⓐ acute angle ⓑ Obtuse angle

T_1 = Force due to surface tension at solid-liquid interface.

T_2 = ——— " ——— solid-air — " —

T_3 = ——— " ——— liquid-air — " —

θ = Angle of contact betⁿ. solid & liquid

From fig ⓐ when the drop is at equilibrium.

$$T_2 = T_1 + T_3 \cos\theta \Rightarrow \therefore \boxed{\cos\theta = \frac{T_2 - T_1}{T_3}}$$

I] If $T_2 > T_1$ & $T_2 - T_1 < T_3$, $\cos\theta < +1$, $\therefore \theta < 90°$ (acute)

II] If $T_2 < T_1$ & $T_1 - T_2 < T_3$, $\cos\theta < -1$, $\therefore \theta > 90°$ (obtuse)

III] If $(T_2 - T_1) = T_3$, $\cos\theta = 1$ $\therefore \theta \approx 0$

IV] If $(T_2 - T_1) > T_3$ or $T_2 > (T_1 + T_3)$, $\cos\theta > 1$ - This is impossible in case of liquid drop.

Effect of Impurity and Temp. on Surface tension.

A] Effect of impurities

1) Highly soluble impurities:
Common salt - dissolved in water ⇒ T - increases ✓

2) Sparingly soluble impurities:
phenol, alcohol, detergent - mixed with water ⇒ T - decreases

3) Insoluble impurities:
$\left.\begin{array}{l}\text{Sand mixed with water}\\ \text{dust gather on mercury}\end{array}\right\} \rightarrow$ T - decreases

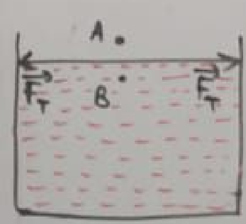
B] Effect of temp:

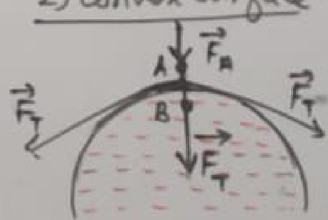
In most of liquids as temp. increases ⇒ T - decreases

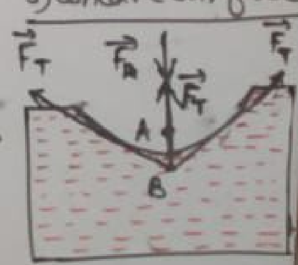
$\left.\begin{array}{l}\text{In molten copper \&}\\ \text{molten cadmium}\end{array}\right\}$ As temp increases ⇒ T - increases ✓

* Excess Pressure across the free surface of liquid

1) Plane Surface 2) Convex Surface 3) Concave Surface







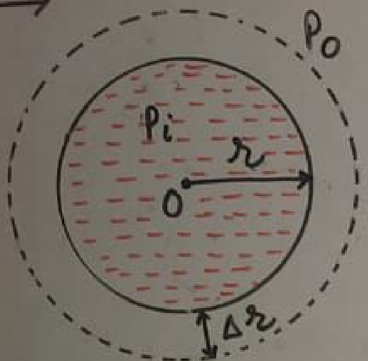
$P_A = P_B$ $P_A < P_B$ $P_A > P_B$

CHAP: 2: Mechanical Properties of Fluids - Part - 5(A)

5

Que: Explain Laplace's law of spherical membrane for a liquid drop & hence for liquid bubble.

$\Rightarrow$



r = Radius of a drop

Δr = Increase in radius of drop

P_i = Inside pressure

P_o = Outside pressure

$(P_i > P_o)$

$\therefore$ Excess pressure inside drop $= (P_i - P_o)$.

Initial surface area of a drop $= A_1 = 4\pi r^2$ —— (1)

Final ———————— $= A_2 = 4\pi(r+\Delta r)^2$

$$\therefore A_2 = 4\pi(r^2 + 2r\Delta r + \Delta r^2)$$

$$\therefore A_2 = 4\pi r^2 + 8\pi r\Delta r + \underline{4\pi\Delta r^2}$$

but Δr is very small $\therefore \Delta r^2$ is negligible

$$\therefore A_2 = 4\pi r^2 + 8\pi r\Delta r \text{ —— (2)}$$

$\therefore$ Increase in surface area of a drop $= dA = A_2 - A_1 = 8\pi r\Delta r$ —— (3)

The workdone in increase in surface area dA is given by

$$dw = T\,dA = T \times 8\pi r\Delta r \text{ —— (4)}$$

The workdone is the product of force 'F' & displacement 'Δr'

$$\therefore dW = F \cdot \Delta r \text{ —— (5)}$$

but excess force is given by $\left(P = \frac{F}{A}\right)$

Excess force = Excess pressure × Surface area of a drop

$$\therefore F = (P_i - P_o) \times 4\pi r^2 \text{ —— (6)}$$

put (6) into (5)

$$\therefore dW = (P_i - P_o) \times 4\pi r^2 \times \Delta r \text{ —— (7)}$$

Equating (4) & (7)

$$\therefore (P_i - P_o) \times \cancel{4}\cancel{\pi} r^{\cancel{2}} \times \cancel{\Delta r} = T \times \cancel{8}^{2}\cancel{\pi}\cancel{r}\cancel{\Delta r}$$

$$\therefore \boxed{(P_i - P_o) = \frac{2T}{r}} \text{ —— (8)}$$

This eq^n gives excess pressure inside liquid drop. This is Laplace's law of spherical membrane.

*** For liquid bubble (soap bubble):**

There are two surfaces of bubble in contact with air

The workdone is

$$\therefore dW = T2 \times 8\pi r\Delta r = 16\pi r\Delta r T$$

$\therefore$ workdone with excess pressure is dw given by air

$$dW = (P_i - P_o) \times \cancel{4}\cancel{\pi} r^{\cancel{2}} \cancel{\Delta r} = \cancel{16}^{4}\cancel{\pi}\cancel{r}\cancel{\Delta r}T$$

$$\therefore \boxed{P_i - P_o = \frac{4T}{r}} \text{ —— (9)}$$

*** Capillary:** It is tube having very fine bore (1mm) & open at both ends.

*** Capillarity:** It is the phenomenon of rise & fall of liquid inside capillary.

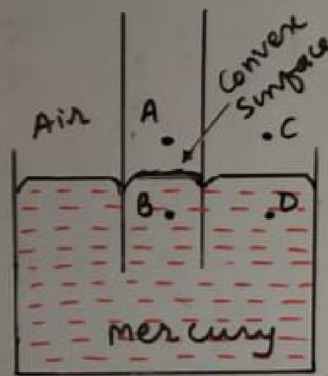
eg. 1) Oil rises up the wick of lamp.

2) Blotting paper absorbs ink.

CHAP: 2: Mechanical Properties OF Fluids - Part - 5(B)

* Action of capillary

I] Capillary Fall: A liquid which does not wets solid (mercury in glass capillary)



(a) Capillary in mercury before drop in level.

A = Just above Convex surface inside Capillary

B = — " — below — " —

C = Just above plane surface outside capillary

D = — " — below — " —

P_A, P_B, P_C & P_D Pressures at A, B, C & D respectively.

From fig (a)

$P_B > P_A$ — ① — (convex surface)

As A & C are in air at same level

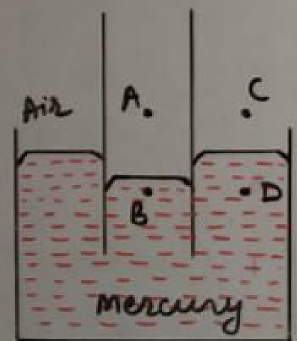
$\therefore P_A = P_C$ — ②

Also $P_C = P_D$ — ③ — (Plane surface)

$\therefore P_A = P_D$ — ④

From ① & ④

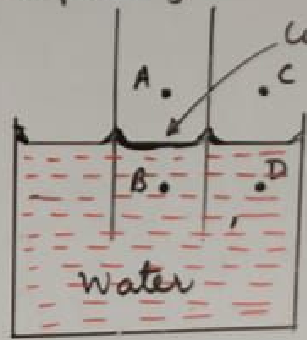
$\boxed{P_B > P_D}$ — ⑤



(b) Capillary in mercury after Fall (drop) in level

As B & D are at same level but at diff pressures. we know liquid flows from high pressure to low pressure So mercury falls in capillary.

II] Capillary rise: A liquid which completely wets solid (water in glass).



$P_B < P_A$ — ① — (concave surface)

$P_A = P_C$ — ② — (At same level in air)

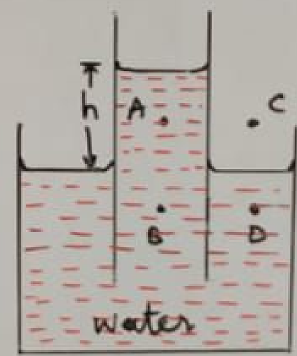
$P_C = P_D$ — ③ — (Plane surface)

$\therefore$ From ② & ③

$P_A = P_D$ — ④

From ① & ④

$\boxed{P_B < P_D}$ — ⑤



As B & D are at same level inside liquid, but at diff pressures. So liquid flows from high pressure to low pressure

So water rises inside capillary.

CHAP: 2: Mechanical Properties of Fluids - Part - 5(B)

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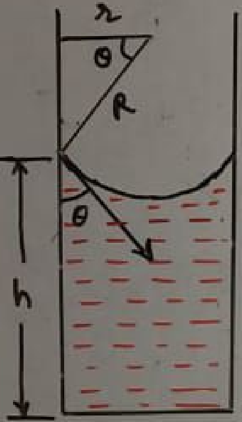
Expression for Capillary rise & fall (surface tension)

I] Using Pressure difference



The pressure due to liquid (water) column of height 'h' must be equal to pressure diff $\left(\frac{2T}{R}\right)$

$$\therefore h\rho g = \frac{2T}{R} \quad \text{—①}$$

h = height of liquid column

ρ = density of liquid

R = Radius of curvature of meniscus.

T = surface tension of liquid.

From fig. r is radius of Capillary tube

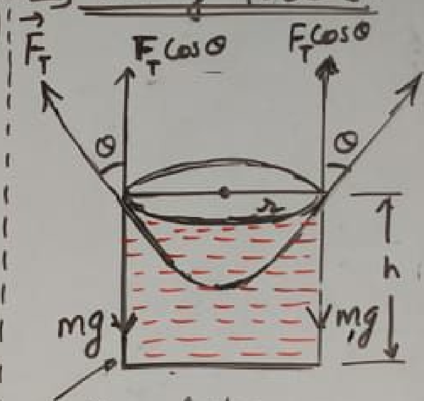
$$\therefore R = \frac{r}{\cos\theta} \quad \text{—②}$$

put ② into ①

$$h\rho g = \frac{2T\cos\theta}{r}$$

$$\therefore \boxed{T = \frac{rh\rho g}{2\cos\theta}} \quad \text{—③}$$

II] Using forces.



$\vec{F}_T$ resolves into two components

1) $F_T\cos\theta$ - vertical upward

2) $F_T\sin\theta$ - horizontal outward

Force due to surface Tension = Surface Tension × circumference

$$F_T = T \times 2\pi r \quad \text{—①}$$

The total upward force $= F_T\cos\theta$ —②

$= T \times 2\pi r \times \cos\theta$ —③

'm' is the mass of liquid column rises in capillary.

$\therefore$ Total downward force $= mg$ —④

but $m = V \times \rho = \pi r^2 h\rho$

$\therefore$ Total downward force $= \pi r^2 h\rho g$ —⑤

At equilibrium position of liquid column

Total upward force = Total downward force

$$T \times 2\cancel{\pi}\cancel{r} \times \cos\theta = \cancel{\pi} r^{\cancel{2}} h\rho g$$

$$\boxed{T = \frac{rh\rho g}{2\cos\theta}} \quad \text{—⑥}$$